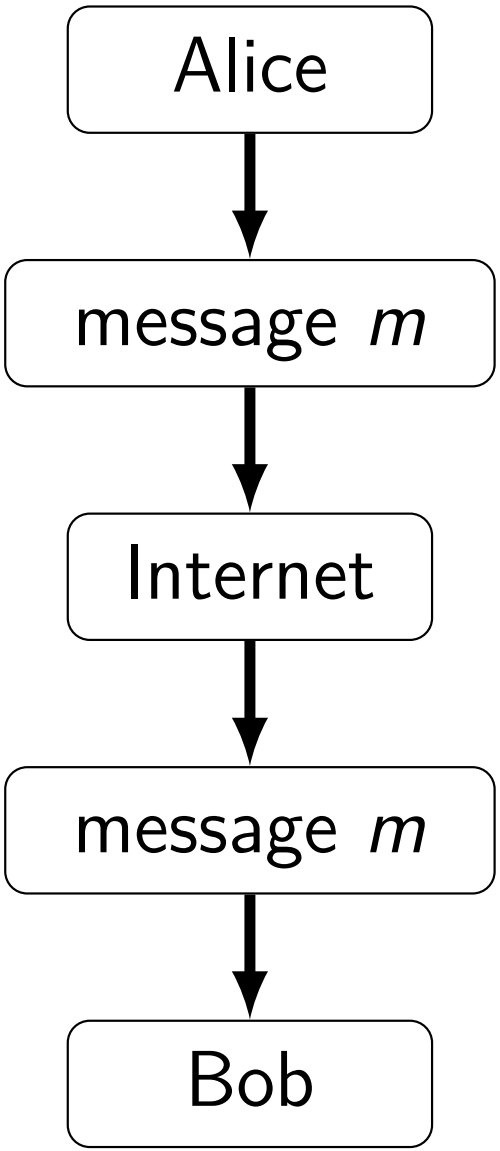
Symmetric cryptanalysis

Daniel J. Bernstein

Attack 0: look for plaintext



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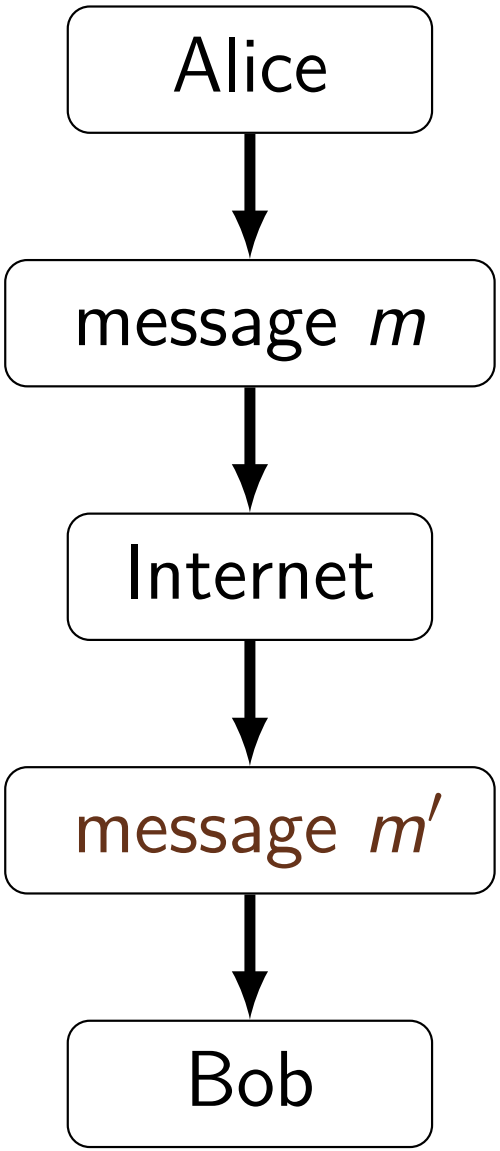

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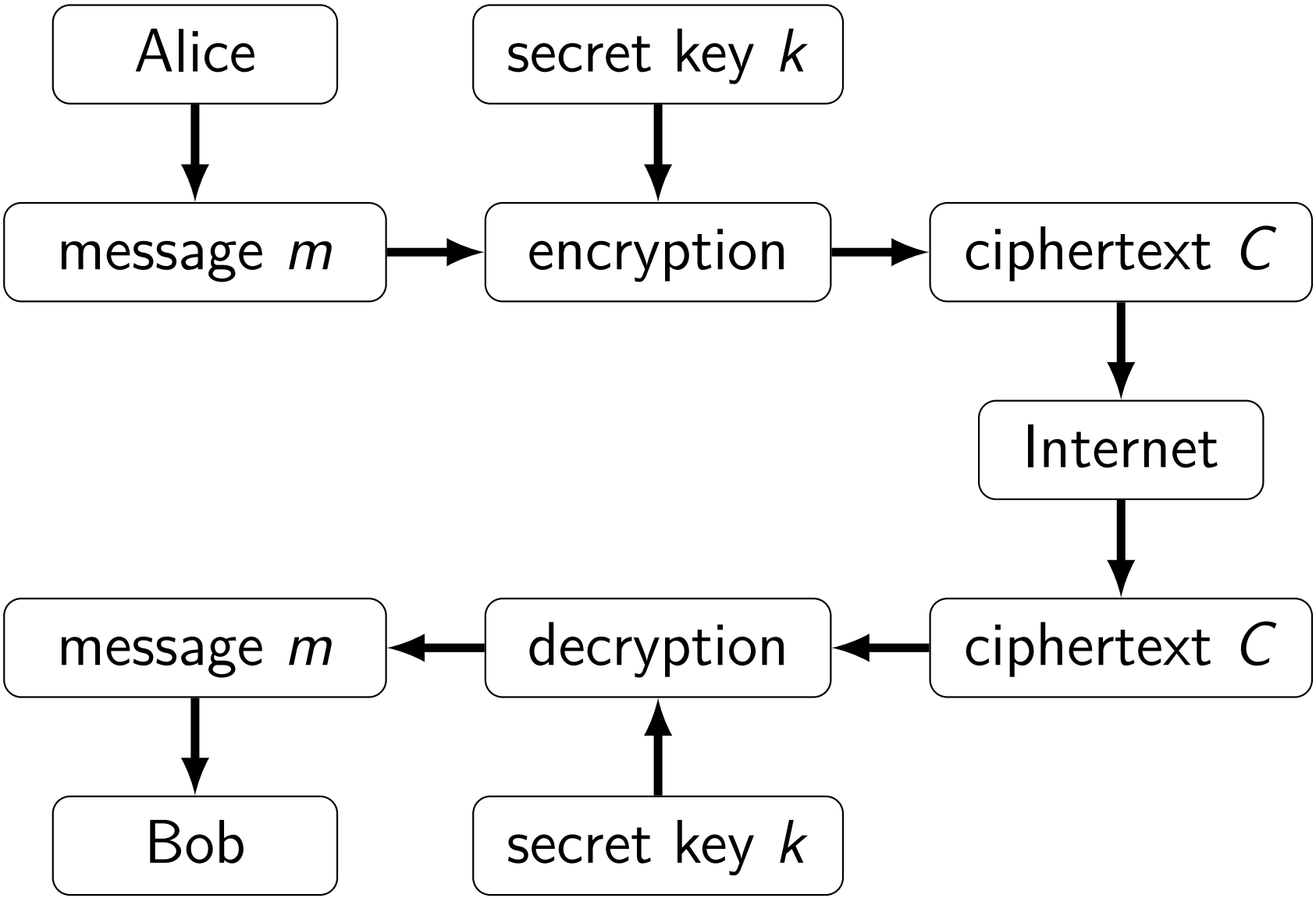
m travels through the Internet.
This is a failure of **confidentiality**.

Can also carry out active attacks:
modify message m into m',
or forge a new message m'.
Bob thinks m' comes from Alice.
This is a failure of **integrity**.

Symmetric cryptography: basic data flow



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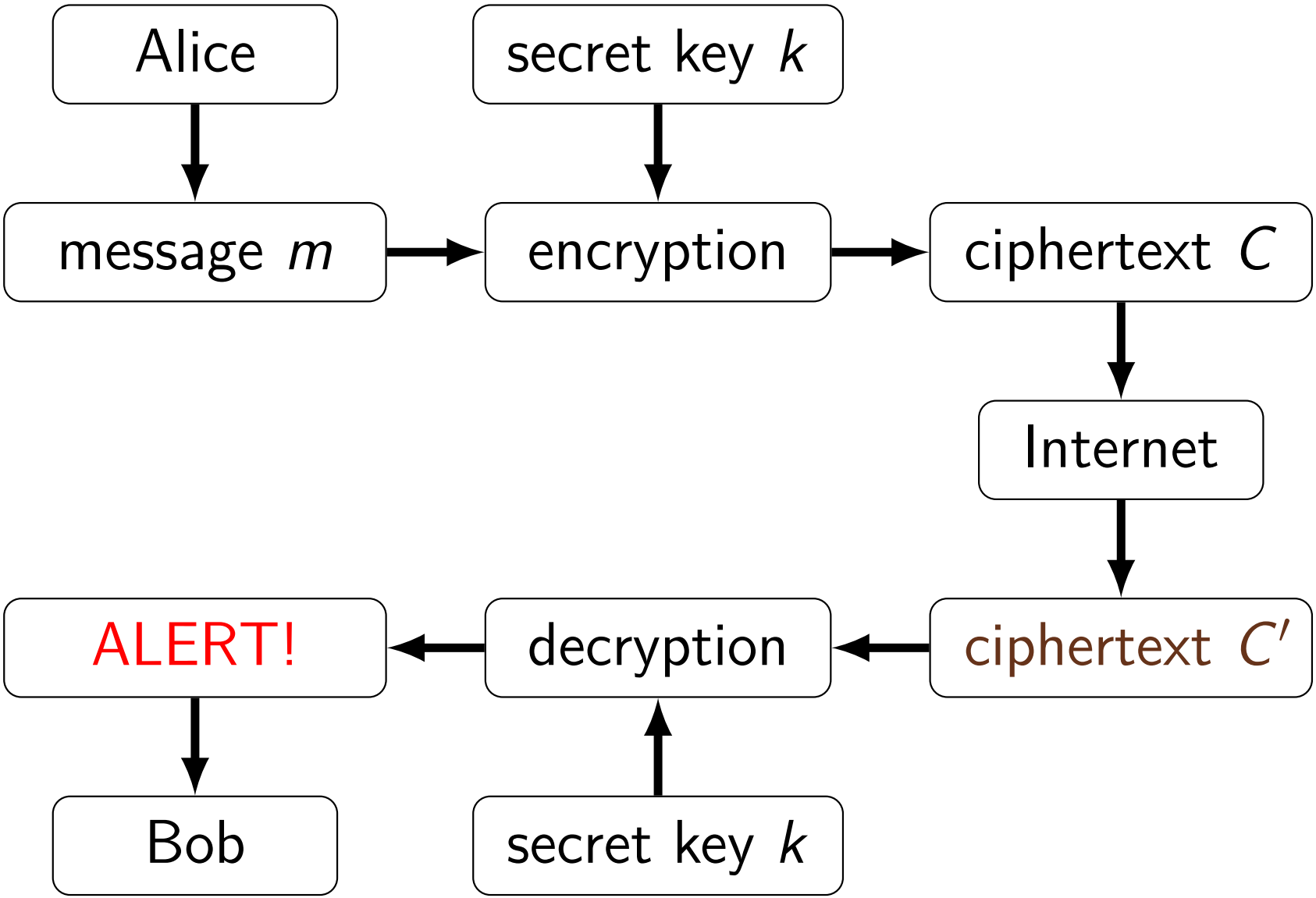

Information-theoretic security

The one-time pad

Say Alice's first message m has 30 digits.

Alice's encryption: $C = (m + k_1) \bmod 10^{30}$
where $k_1 = k \bmod 10^{30}$.

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Bob's decryption: $m = (C - k_1) \bmod 10^{30}$.

"One-time": Alice+Bob burn k_1 after it is used;
i.e., Alice+Bob replace k with $\lfloor k/10^{30} \rfloor$.
Next message uses next digits k_2 from k.

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Example of technique to try to hide metadata: "mixnets". Won't say more about this today.

Message-authentication codes (MACs)

The one-time pad doesn't stop attacker from modifying messages.
e.g. attacker adds 1 to C; this adds 1 to m.

Message authentication protects integrity:

- Alice uses more digits of secret key to compute an **authenticator** of C.
- Alice sends the authenticator along with C.
- Bob recomputes the authenticator.
- Bob rejects C if authenticator doesn't match.

A simple MAC

Standardize a prime $p = 1000003$.

To authenticate a 30-digit message
$(C_1, C_2, C_3, C_4, C_5)$
where each $C_i \in \{0, 1, \ldots, 999999\}$,
Alice sends 6-digit authenticator
$(((C_1 r_1 + \cdots + C_5 r_5) \bmod p) + s) \bmod 1000000$,
using 36 digits from secret key:
$r_1, r_2, r_3, r_4, r_5, s \in \{0, 1, \ldots, 999999\}$.

Example 1

$r_1 = 314159$, $r_2 = 265358$, $r_3 = 979323$,
$r_4 = 846264$, $r_5 = 338327$, $s = 950288$,
$C = 000006\,000007\,000000\,000000\,000000$:

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Alice computes authenticator
$(6r_1 + 7r_2 \bmod p) + s \bmod 1000000 =$
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Alice transmits
$000006\,000007\,000000\,000000\,000000\,692739$.

A MAC using fewer secret digits

Alice sends 6-digit authenticator
$(C_1 r + C_2 r^2 + \cdots + C_5 r^5 \bmod p) + s \bmod 1000000$,
using 12 digits from secret key:
$r, s \in \{0, 1, \ldots, 999999\}$.

i.e.: take $r_i = r^i$ in previous authenticator,
replacing separate 6-digit secrets $r_1, r_2, \ldots$
with just one 6-digit secret r.

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Sender transmits authenticated message
$000006\,000007\,000000\,000000\,000000\,218669$.

Is this secure?

Attacker sees C, a.

Attacker's goal: Find C', a' such that

- $a' = (C'(r) \bmod p) + s \bmod 1000000$ and
- $C' \neq C$.

Here $C'(r) = C'_1 r + C'_2 r^2 + C'_3 r^3 + C'_4 r^4 + C'_5 r^5$.

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Can repeat attack. Each forgery has chance $1/1000000$ of being accepted.

A more powerful attack

Choose $a' = a$. Choose $C' \neq C$ so that the polynomial $C'(x) - C(x)$ has 5 distinct roots $x \in \{0, 1, \ldots, 999999\}$ modulo p.

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e.g. $C = (100, 0, 0, 0, 0)$, $C' = (125, 1, 0, 0, 1)$: $C'(x) - C(x) = x^5 + x^2 + 25x$ which has five roots mod p, namely $0, 299012, 334447, 631403, 735144$.

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Success chance $5/1000000$.

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Example: If $C(334885) \bmod p$
$\in \{1000000, 1000001, 1000002\}$
then a forgery (C', a') with
$C'(x) = C(x) + x^5 + x^2 + 25x$ and $a' = a$
succeeds not just for
$r \in \{0, 299012, 334447, 631403, 735144\}$
from previous slide, but also for $r = 334885$,
which is a root of $C'(x) - C(x) + 1000000$.

Attacker can't do much better than this

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Underlying fact: the polynomial
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Warning: very easy to break the simpler variant
$(C_1 + C_2 r + \cdots + C_5 r^4 \bmod p) + s \bmod 1000000$;
simply solve $C'(x) - C(x) = a' - a$.

Scale up for serious security

Poly1305 is a MAC that uses 128-bit r's,
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e.g. 2^{64} forgeries, $L = 1536$:
Pr[all rejected] ≥ 0.9999999998.

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Alice and Bob can reuse r for many messages.
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e.g. Poly1305: Split string into 16-byte chunks, maybe with smaller final chunk; append 1 to each chunk; view as little-endian integers in $\{1, 2, 3, \ldots, 2^{129}\}$. Multiply first chunk by r, add next chunk, multiply by r, etc., add last chunk, multiply by r, mod $2^{130} - 5$, add s mod 2^{128}.

Is this safe in the real world?

Maybe the secret key isn't truly random. Example: SmartFacts exploited bad randomness-generation hardware in smart cards that had NIST/CSE FIPS certification and BSI Common Criteria certification.

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Maybe an attacker corrupted the random-number generator. See, e.g., NSA's backdoored Dual EC.

Symmetric vs. public-key

Spying on the initial key exchange

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Public-key encryption tries to solve this,
allowing Alice and Bob to share secret k
through a public communication channel.
Many proposed public-key cryptosystems are broken,
but details are outside today's scope.

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This split also means that a single public-key operation handles any number of messages from Alice to Bob, so less incentive for developers to scale down public-key security levels.

Ciphers

The key-size mismatch

Public-key crypto typically shares a short secret key: e.g., Alice and Bob end up sharing a 256-bit k.

The one-time pad uses 1MB of secrets to encrypt 1MB of user data. Also, a polynomial MAC needs a new s for each message. How do Alice and Bob share all this secret data?

Using ciphers to expand keys

AES expands 256-bit secret k into many secrets $F(k,1), F(k,2), F(k,3), \ldots$; AES-GCM uses these for the one-time pad and a polynomial MAC.

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Definition of **PRG** ("pseudorandom generator"): Attacker can't distinguish $F(k,1), F(k,2), F(k,3), \ldots$ from string of independent uniform random blocks.

PRF, PRP

PRF ("pseudorandom function"):
Attacker can't distinguish
$F(k,1), F(k,2), F(k,3), \ldots$ from independent uniform random blocks, given access to a server that returns $F(k,i)$ given i. Server is called an **oracle**.

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If block size is big then PRP $\Rightarrow$ PRF $\Rightarrow$ PRG.

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ChaCha20 block size: 512 bits.

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Can prove confidentiality and integrity
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(aka **mode of operation**) of F.

Check what security features a mode is promising:
e.g., CTR ("counter mode") is good for encryption
but does not try to provide authentication.

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More on proofs

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Open question: What % of "proofs" are wrong?

Sometimes ciphers are not secure

Take a correct proof that $M(F)$ is secure *assuming* F is a PRF. How do we know that AES and ChaCha20 are PRFs?

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Remaining slides today:

- Simple example of block cipher: 1994 "TEA, a tiny encryption algorithm" (Wheeler–Needham). Seems to be a good cipher, *except* that block size is too small: good only for PRP, not PRF.
- Variants of this block cipher that look similar but can be quickly broken.

TEA

Notation

`uint32`: 32 bits $(b_0, b_1, \ldots, b_{31})$ representing the "unsigned" integer $b_0 + 2b_1 + \cdots + 2^{31}b_{31}$.

`+`: addition mod 2^{32}.

`c += d`: same as `c = c + d`.

`^`: xor; $\oplus$; addition of each bit separately mod 2. Lower precedence than + in C.

`<<4`: mult by 16, i.e., $(0, 0, 0, 0, b_0, b_1, \ldots, b_{27})$.

`>>5`: div by 32, i.e., $(b_5, b_6, \ldots, b_{31}, 0, 0, 0, 0, 0)$.

TEA

```
void encrypt(uint32 *b,uint32 *k) {
  uint32 x = b[0], y = b[1];
  uint32 r, c = 0;
  for (r = 0;r < 32;r += 1) {
    c += 0x9e3779b9;
    x += y+c ^ (y<<4)+k[0] ^ (y>>5)+k[1];
    y += x+c ^ (x<<4)+k[2] ^ (x>>5)+k[3];
  }
  b[0] = x; b[1] = y;
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Input: 128-bit key (`k[0]`,`k[1]`,`k[2]`,`k[3]`);
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Output: 64-bit **ciphertext** (final `b[0]`,`b[1]`).

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64-bit **plaintext** (`b[0]`,`b[1]`).

Output: 64-bit **ciphertext** (final `b[0]`,`b[1]`).

Can quickly **encrypt**: (key, plaintext) $\mapsto$ ciphertext.

Can quickly **decrypt**: (key, ciphertext) $\mapsto$ plaintext.

Wait, how can we decrypt?

```
void encrypt(uint32 *b,uint32 *k) {
  uint32 x = b[0], y = b[1];
  uint32 r, c = 0;
  for (r = 0;r < 32;r += 1) {
    c += 0x9e3779b9;
    x += y+c ^ (y<<4)+k[0] ^ (y>>5)+k[1];
    y += x+c ^ (x<<4)+k[2] ^ (x>>5)+k[3];
  }
  b[0] = x; b[1] = y;
}
```

Answer: Each step is invertible

```
void decrypt(uint32 *b,uint32 *k) {
  uint32 x = b[0], y = b[1];
  uint32 r, c = 32 * 0x9e3779b9;
  for (r = 0;r < 32;r += 1) {
    y -= x+c ^ (x<<4)+k[2] ^ (x>>5)+k[3];
    x -= y+c ^ (y<<4)+k[0] ^ (y>>5)+k[1];
    c -= 0x9e3779b9;
  }
  b[0] = x; b[1] = y;
}
```

Generalization: "Feistel network"

(used in, e.g., 1973 Feistel–Coppersmith "Lucifer")

```
x += function1(y,k); y += function2(x,k);
x += function3(y,k); y += function4(x,k);
...
```

Decryption, inverting each step:

```
...
y -= function4(x,k); x -= function3(y,k);
y -= function2(x,k); x -= function1(y,k);
```

Do we need decryption?

If F is used only to expand key k into $F(k,1), F(k,2), \ldots$ then no need to invert F: e.g., encrypt block by adding $F(k,1)$, decrypt block by subtracting $F(k,1)$.

But some modes apply F to blocks of user data, and need F^{-1} for decryption. Literature sometimes claims that these modes have advantages.

TEA again

```
void encrypt(uint32 *b,uint32 *k) {
  uint32 x = b[0], y = b[1];
  uint32 r, c = 0;
  for (r = 0;r < 32;r += 1) {
    c += 0x9e3779b9;
    x += y+c ^ (y<<4)+k[0] ^ (y>>5)+k[1];
    y += x+c ^ (x<<4)+k[2] ^ (x>>5)+k[3];
  }
  b[0] = x; b[1] = y;
}
```

XORTEA: a bad cipher

```
void encrypt(uint32 *b,uint32 *k) {
  uint32 x = b[0], y = b[1];
  uint32 r, c = 0;
  for (r = 0;r < 32;r += 1) {
    c += 0x9e3779b9;
    x ^= y^c ^ (y<<4)^k[0] ^ (y>>5)^k[1];
    y ^= x^c ^ (x<<4)^k[2] ^ (x>>5)^k[3];
  }
  b[0] = x; b[1] = y;
}
```

XORTEA vs. TEA

"Hardware-friendlier" cipher,
since xor circuit is cheaper than add.

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But output bits are linear functions of input bits!

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But output bits are linear functions of input bits!

e.g. First output bit is
$1 \oplus k_0 \oplus k_1 \oplus k_3 \oplus k_{10} \oplus k_{11} \oplus k_{12} \oplus k_{20} \oplus k_{21} \oplus k_{30} \oplus k_{32} \oplus$
$k_{33} \oplus k_{35} \oplus k_{42} \oplus k_{43} \oplus k_{44} \oplus k_{52} \oplus k_{53} \oplus k_{62} \oplus k_{64} \oplus k_{67} \oplus$
$k_{69} \oplus k_{76} \oplus k_{85} \oplus k_{94} \oplus k_{96} \oplus k_{99} \oplus k_{101} \oplus k_{108} \oplus k_{117} \oplus$
$k_{126} \oplus b_1 \oplus b_3 \oplus b_{10} \oplus b_{12} \oplus b_{21} \oplus b_{30} \oplus b_{32} \oplus b_{33} \oplus b_{35} \oplus$
$b_{37} \oplus b_{39} \oplus b_{42} \oplus b_{43} \oplus b_{44} \oplus b_{47} \oplus b_{52} \oplus b_{53} \oplus b_{57} \oplus b_{62}$.

Turning linearity into an attack

There is a matrix M with coefficients in $\mathbb{F}_2$
such that $\mathrm{XORTEA}_k(b) = (1, k, b)M$ for all (k, b).

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Very fast attack: if $b_4 = b_1 \oplus b_2 \oplus b_3$ then $\mathrm{XORTEA}_k(b_1) \oplus \mathrm{XORTEA}_k(b_2) = \mathrm{XORTEA}_k(b_3) \oplus \mathrm{XORTEA}_k(b_4)$.

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$\mathrm{XORTEA}_k(b_1) \oplus \mathrm{XORTEA}_k(b_2) =$
$\mathrm{XORTEA}_k(b_3) \oplus \mathrm{XORTEA}_k(b_4)$.

This breaks PRP (and PRF):
uniform random permutation (or function) F
almost never has $F(b_1) \oplus F(b_2) = F(b_3) \oplus F(b_4)$.

TEA again

```
void encrypt(uint32 *b,uint32 *k) {
  uint32 x = b[0], y = b[1];
  uint32 r, c = 0;
  for (r = 0;r < 32;r += 1) {
    c += 0x9e3779b9;
    x += y+c ^ (y<<4)+k[0] ^ (y>>5)+k[1];
    y += x+c ^ (x<<4)+k[2] ^ (x>>5)+k[3];
  }
  b[0] = x; b[1] = y;
}
```

LEFTEA: another bad cipher

```
void encrypt(uint32 *b,uint32 *k) {
  uint32 x = b[0], y = b[1];
  uint32 r, c = 0;
  for (r = 0;r < 32;r += 1) {
    c += 0x9e3779b9;
    x += y+c ^ (y<<4)+k[0] ^ (y<<5)+k[1];
    y += x+c ^ (x<<4)+k[2] ^ (x<<5)+k[3];
  }
  b[0] = x; b[1] = y;
}
```

Breaking LEFTEA

Addition is not $\mathbb{F}_2$-linear,
but addition mod 2 is $\mathbb{F}_2$-linear.

First output bit is $1 \oplus k_0 \oplus k_{32} \oplus k_{64} \oplus k_{96} \oplus b_{32}$.

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but they never affect first bit.

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In TEA, >>5 **diffuses** nonlinear changes
from high bits to low bits. LEFTEA misses this.

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In TEA, >>5 **diffuses** nonlinear changes
from high bits to low bits. LEFTEA misses this.

(LEFTEA, like TEA, diffuses changes from low bits
to high bits via carries in addition and via <<4.)

TEA again

```
void encrypt(uint32 *b,uint32 *k) {
  uint32 x = b[0], y = b[1];
  uint32 r, c = 0;
  for (r = 0;r < 32;r += 1) {
    c += 0x9e3779b9;
    x += y+c ^ (y<<4)+k[0] ^ (y>>5)+k[1];
    y += x+c ^ (x<<4)+k[2] ^ (x>>5)+k[3];
  }
  b[0] = x; b[1] = y;
}
```

TEA4: another bad cipher

```
void encrypt(uint32 *b,uint32 *k) {
  uint32 x = b[0], y = b[1];
  uint32 r, c = 0;
  for (r = 0;r < 4;r += 1) {
    c += 0x9e3779b9;
    x += y+c ^ (y<<4)+k[0] ^ (y>>5)+k[1];
    y += x+c ^ (x<<4)+k[2] ^ (x>>5)+k[3];
  }
  b[0] = x; b[1] = y;
}
```

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Fast attack:
$\mathrm{TEA4}_k(x + 2^{31}, y)$, $\mathrm{TEA4}_k(x, y)$ have same first bit.

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(Why? Trace x, y differences through computation.
$r = 0$: multiples of $2^{31}, 2^{26}$.
$r = 1$: multiples of $2^{21}, 2^{16}$.
$r = 2$: multiples of $2^{11}, 2^{6}$.
$r = 3$: multiples of $2^{1}, 2^{0}$.)

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Uniform random function F: probability $1/2$ that $F(x + 2^{31}, y)$ and $F(x, y)$ have same first bit.

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$r = 3$: multiples of $2^{1}, 2^{0}$.)

Uniform random function F: probability $1/2$ that $F(x + 2^{31}, y)$ and $F(x, y)$ have same first bit.

This attack has PRF advantage $1/2$.

Differential attacks, linear attacks, etc.

More sophisticated attacks:
trace *probabilities* of differences;
probabilities of linear equations;
probabilities of higher-order differences
$C(x+\delta+\epsilon)-C(x+\delta)-C(x+\epsilon)+C(x)$; etc.
Use algebra and statistics to exploit
non-randomness in probabilities.

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but rapidly lose effectiveness as r increases.

Hard question in cipher design:
How many "rounds" are really needed for security?

TEA again

```
void encrypt(uint32 *b,uint32 *k) {
  uint32 x = b[0], y = b[1];
  uint32 r, c = 0;
  for (r = 0;r < 32;r += 1) {
    c += 0x9e3779b9;
    x += y+c ^ (y<<4)+k[0] ^ (y>>5)+k[1];
    y += x+c ^ (x<<4)+k[2] ^ (x>>5)+k[3];
  }
  b[0] = x; b[1] = y;
}
```

REPTEA: another bad cipher

```
void encrypt(uint32 *b,uint32 *k) {
  uint32 x = b[0], y = b[1];
  uint32 r, c = 0x9e3779b9;
  for (r = 0;r < 1000;r += 1) {

    x += y+c ^ (y<<4)+k[0] ^ (y>>5)+k[1];
    y += x+c ^ (x<<4)+k[2] ^ (x>>5)+k[3];
  }
  b[0] = x; b[1] = y;
}
```

Breaking REPTEA

$\mathrm{REPTEA}_k(b) = R_k^{1000}(b)$
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Try list of 2^{32} inputs b.
Collect outputs $\mathrm{REPTEA}_k(b)$.

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Good chance that some b in list also has $a = R_k(b)$ in list. Then $\mathrm{REPTEA}_k(a) = R_k(\mathrm{REPTEA}_k(b))$.

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Good chance that some b in list also has $a = R_k(b)$ in list. Then $\mathrm{REPTEA}_k(a) = R_k(\mathrm{REPTEA}_k(b))$.

For each (b, a) from list: Try solving equations $a = R_k(b)$ and $\mathrm{REPTEA}_k(a) = R_k(\mathrm{REPTEA}_k(b))$ to figure out k. (Exercise: Obtain more equations.)

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This is an example of a **slide attack.**
TEA avoids this by varying c.

Is original TEA secure?

```
void encrypt(uint32 *b,uint32 *k) {
  uint32 x = b[0], y = b[1];
  uint32 r, c = 0;
  for (r = 0;r < 32;r += 1) {
    c += 0x9e3779b9;
    x += y+c ^ (y<<4)+k[0] ^ (y>>5)+k[1];
    y += x+c ^ (x<<4)+k[2] ^ (x>>5)+k[3];
  }
  b[0] = x; b[1] = y;
}
```

A related-key property

Related keys: e.g., $\mathrm{TEA}_{k'}(b) = \mathrm{TEA}_k(b)$ where $(k'[0], k'[1], k'[2], k'[3]) =$
$(k[0] + 2^{31}, k[1] + 2^{31}, k[2], k[3])$.

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PRP attack goal: distinguish TEA_k, for one secret key k, from uniform random permutation.

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PRP attack goal: distinguish TEA_k, for one secret key k, from uniform random permutation.

Brute-force attack: Guess key g;
see if TEA_g matches TEA_k on some outputs.

Related keys $\Rightarrow$ g succeeds with chance 2^{-126}.
Still very small.

More related-key properties

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Fancier relationship between k, k' has chance 2^{-11}
of producing a particular output equation.

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No evidence in literature that this helps brute-force attack, or otherwise affects PRP security. No challenge to security analysis of modes using TEA.

But advertised as "related-key cryptanalysis" and claimed to justify recommendations for designers regarding key scheduling.

Further reading

Some ways to learn more about cipher attacks, hash-function attacks, etc.:

- Read attack papers. Some useful sources:
  eSTREAM competition (2004–2008);
  SHA-3 competition (2007–2012);
  CAESAR competition (2013–2019);
  NISTLWC competition (2019–2023);
  FSE conference.
- Try to break ciphers yourself:
  e.g., find attacks on FEAL or on RC4.
  Reasonable starting point: 2000 Schneier
  "Self-study course in block-cipher cryptanalysis".

Bonus slide: reported ChaCha attack costs